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FURTHER CONSIDERATIONS IN
THE DETECTION OF PULSED
SIGNALS IN NOISE

Report R-30

30 January 1953

Copy_____ of 125 copies

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Contract DA-11-022-ORD-721

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Q # 2134 Q/2L



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Abstract

This report contains corrections to an earlier report (Control Systems Laboratory Report R-26).

The effect of finite signal duration on the behavior of a low pass filter (or exponential integrator) is re-examined. Correction is made for the residue of the noise samples which is present along with the signal plus noise samples whenever the detection system is processing continuous data.

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In part IV of a recent report^{*} use was made of the concept of the "effective integrating time of a low pass filter." Subsequent consideration has shown that although this concept is useful when processing noise, or signals plus noise samples whose duration, T , is much greater than the low pass filter time constant T_0 , the concept is not useful and actually leads to erroneous results when T is finite, particularly when $T < T_0$.

Consider an exponential integrator whose time constant is T_0 and which is summing samples spaced t_1 seconds apart. The signal plus noise samples have a mean \bar{y}_{S+N} and a dispersion $\sigma^2_{y_{S+N}}$. At the end of a signal of duration T , the integrator has stored T/t_1 signal plus noise samples and an infinite number of noise samples of mean \bar{y}_N and dispersion $\sigma^2_{y_N}$. These samples are all exponentially weighted so that even the infinite number of noise samples make only a finite contribution to the sum. The error in the previous paper consisted in not allowing for the presence of this residue of noise samples which is always

* C. W. Sherwin, "The Detection of Pulsed Signals in Noise" Report R-26, Control Systems Laboratory, University of Illinois, September 1952.

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present when the signal has finite duration.

Let \bar{z}_{S+N} = the integrator output at the end of a signal of duration T . Since t_i = the spacing of the independent samples, $n_i = \frac{1}{t_i}$ = the rate at which independent samples are entering the integrator.

$$\bar{z}_{S+N} = \bar{y}_{S+N} \left(1 + e^{-t_i/T_0} + e^{-2t_i/T_0} + \dots + e^{-(n-1)t_i/T_0} \right) + \bar{y}_N \left(e^{-nt_i/T_0} + e^{-(n+1)t_i/T_0} + \dots \right)$$

Using the identity $1 + x + x^2 + \dots + x^{n-1} = \frac{1-x^n}{1-x}$, also let $n = \frac{T}{t_i}$

$$\bar{z}_{S+N} = \frac{\bar{y}_{S+N} (1 - e^{-T/T_0}) + \bar{y}_N e^{-T/T_0}}{1 + e^{-1/n_i T_0}} \quad (1)$$

Similarly,

$$\begin{aligned} \sigma_{\bar{z}_{S+N}}^2 &= \sigma_{\bar{y}_{S+N}}^2 \left(1 + e^{-2t_i/T_0} + e^{-2 \cdot 2t_i/T_0} + \dots + e^{-2(n-1)t_i/T_0} \right) \\ &\quad + \sigma_{\bar{y}_N}^2 \left(e^{-2nt_i/T_0} + e^{-2(n+1)t_i/T_0} + \dots \right) \\ \sigma_{\bar{z}_{S+N}}^2 &= \frac{\sigma_{\bar{y}_{S+N}}^2 (1 - e^{-2T/T_0}) + \sigma_{\bar{y}_N}^2 e^{-2T/T_0}}{1 - e^{-2/n_i T_0}} \quad (2) \end{aligned}$$

When $n_1 T_0 \gg 1$ there are many independent samples being summed and (1) and (2) are adequate to characterize the gaussian output probability distribution of the integrator.

Note that for signals of long duration ($T \gg T_0$), (1) and (2) become

$$\bar{z}_{S+N} = \eta_i T_0 \bar{y}_{S+N} \quad (3)$$

and

$$\sigma_{z_{S+N}}^2 = \frac{\eta_i T_0}{2} \sigma_{y_{S+N}}^2 \quad (4)$$

The ratio of \bar{z}/σ_z is the same as for a linear integrator which sums for a time $2T_0$, the "equivalent integration time". However, for finite T one needs to work directly with (1) and (2) in order calculate the mean and the dispersion of the output of the exponential integrator.

Equations (3) and (4) also apply to noise if its duration is long compared to T_0 . Thus the mean of the output distribution for noise input is

$$\bar{z}_N = \eta_i T_0 \bar{y}_N \quad (5)$$

and the standard deviation of the noise alone distribution is

$$\sigma_{z_N} = \sqrt{\frac{\eta_i T_0}{2}} \sigma_{y_N} \quad (6)$$

If a low false alarm rate is required, the alarm threshold must be several (K_N) standard deviations higher than \bar{z}_N . Thus

$$z_{\text{Threshold}} = \eta_i T_0 \bar{y}_N + K_N \sqrt{\frac{\eta_i T_0}{2}} \sigma_{y_N} \quad (7)$$

Finally if a signal is to be detected 50% of the time, its mean must equal the threshold.

$$z_{\text{Threshold}} = \bar{z}_{s+N} = \eta_i T_0 \left[\bar{y}_{s+N} (1 - 2^{-T/T_0}) + \bar{y}_N 2^{-T/T_0} \right] \quad (8)$$

Equating (7) and (8), we obtain 50% signal detection if

$$(\bar{y}_{s+N})_{50\%} = \bar{y}_N + K_N \frac{1}{\sqrt{2\eta_i T_0}} \cdot \frac{1}{(1 - 2^{-T/T_0})} \cdot \sigma_{y_N} \quad (9)$$

The type of detector determines \bar{y}_N and σ_{y_N} (see Figs. 1 and 2 of original report), and also the manner in

which \bar{y}_{S+N} and $\sigma_{y_{S+N}}$ vary with input signal to noise power. K_N is determined by the required false alarm rate, and n_i by the shape of the noise band (Table I in original report). Thus the value of \bar{y}_{S+N} needed to attain 50% detection can be calculated and then the corresponding $\sigma_{y_{S+N}}$ and input signal to noise power can be read off Figs. 1 and 2.

Since $(\bar{y}_{S+N})_{50\%}$ in equ. (9) does not involve knowledge of $\sigma_{y_{S+N}}$, it can be easily calculated. If however, one wishes a higher signal detectability one needs also to know $\sigma_{y_{S+N}}$. For example for 84% signal detection one needs a value of \bar{z}_{S+N} which is one unit of σ_z above the threshold. Let K_S be the number of units of σ_z by which \bar{z}_{S+N} exceeds the threshold, then instead of equ. (9) we have

$$\bar{y}_{S+N} = \bar{y}_N + \frac{1}{\sqrt{2n_i T_0}} \cdot \frac{1}{(1 - 2^{-T/T_0})} \left[K_N \sigma_{y_N} + K_S \left\{ \sigma_{y_{S+N}}^2 (1 - 2^{-2T/T_0}) + \sigma_{y_N}^2 2^{-2T/T_0} \right\}^{1/2} \right] \quad (10)$$

In (10), \bar{y}_{S+N} and $\sigma_{y_{S+N}}$ are both unknown. A quickly converging solution can be made however by first using (9) to get the value of \bar{y}_{S+N} needed for 50% detection and then using the value of $\sigma_{y_{S+N}}$ thus determined in equ. (10), to give a second value of \bar{y}_{S+N} and also $\sigma_{y_{S+N}}$. This last value of $\sigma_{y_{S+N}}$ can again be substituted into (10) etc. The relative insensitivity of (10) to the value of $\sigma_{y_{S+N}}$ makes this process converge rapidly. It is in fact practically necessary due to the lack of a simple analytical relationship between $\sigma_{y_{S+N}}$ and \bar{y}_{S+N} .

An Example Related to Hearing

Consider a signal of duration $T = 1.0$ second, a square pass band of 60 cps, (thus $n_1 = 60/\text{second}$) a square law detector and an exponential integrator of time constant $T_0 = 0.5$ second.

Require that the false alarm rate be one in 200 seconds and since the filter reaches an independent value at a rate of $2T_0/\text{second}$ the false alarm probability is 5×10^{-3} . This requires a threshold at $K_N = 2.6$ units of σ_{zN} above the mean value of the noise \bar{z}_N . From (9)

since from Fig. 2 both \bar{y}_N and $\sigma_{y_N}^2 = 2\sigma^2$ for the square law detector. From Fig. 2 we find that $P^2/2\sigma^2 = 0.38$.

Psycho acoustic measurements indicate that the human ear plus brain achieves the above specified detection state when the signal power has about .7 to 1.0 times the noise power in the critical band (which is approximately 60 cps wide at frequencies of a few hundred cps.). Thus it appears that the filter-integrator system analyzed above should detect signals about 3 db below the signal level needed for the human ear plus brain.